QUIZ 9 SOLUTIONS: LESSON 10
SEPTEMBER 19, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

A corporation is initially worth 1 million dollars. Let $V(t)$ denote the value of the company after t years. Suppose that V is growing in value by 20% each year and gaining 15% of a growing market estimated at $100e^{.2t}$ million dollars.

1. [3 pts] Write down a differential equation that describes the change in the value of the company.

__Solution__: We are looking for an equation for $\frac{dV}{dt}$. We are told that V is increasing by 20% each year **and** gaining 15% of an emerging market. This "and" will be represented by a "+". We write

$$\frac{dV}{dt} = .2V + .15\left(100e^{.2t}\right) = \boxed{.2V + 15e^{.2t}}$$

2. [5 pts] Find a general solution to the differential equation described in # 1.

__Solution__: We observe that

$$\frac{dV}{dt} = .2V + 15e^{.2t}$$

is a FOLDE but not *quite* in the correct form. We slightly rewrite to get

$$\frac{dV}{dt} - .2V = 15e^{.2t}.$$

Now, we may go through our steps.

Step 1: Find P, Q

$P(t) = -.2, \quad Q(t) = 15e^{.2t}$

Step 2: Find the integrating factor

$$\begin{aligned} u(t) &= e^{\int P(t)\,dt} \\ &= e^{\int(-.2)\,dt} \\ &= e^{-.2t} \end{aligned}$$

Step 3: Set up solution

$$\begin{aligned}
V \cdot u(t) &= \int Q(t)u(t)\,dt \\
\Rightarrow \quad V\underbrace{(e^{-.2t})}_{u(t)} &= \int \underbrace{(15e^{.2t})}_{Q(t)}\underbrace{(e^{-.2t})}_{u(t)}\,dt \\
&= \int 15e^{.2t}e^{-.2t}\,dt \\
&= \int 15e^{.2t-.2t}\,dt \\
&= \int 15\underbrace{e^{0}}_{1}\,dt \\
&= \int 15\,dt \\
\Rightarrow \quad Ve^{-.2t} &= 15t + C \\
\Rightarrow \quad V &= \frac{15t+C}{e^{-.2t}} = \boxed{e^{.2t}(15t+C)}
\end{aligned}$$

3. [2 pts] Find the value of the company after 10 years (round your answer to the nearest million).

Solution: In # 2, we computed the general solution to our differential equation. We now use the fact that $V(0) = 1$ to solve for C. Write

$$\begin{aligned}
\underbrace{1}_{V(0)} &= e^{.2\cdot 0}(15 \cdot 0 + C) \\
&= 1 \cdot (0 + C) = C \\
\Rightarrow \quad 1 &= C
\end{aligned}$$

Thus,

$$V(t) = e^{.2t}(15t + 1).$$

Finally, we write

$$V(10) = e^{.2(10)}(15(10) + 1) \approx \boxed{1116 \text{ million dollars}}$$